

## Testing the Maldacena conjecture with SDLCQ

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We report on a test of the Maldacena conjecture<sup>1</sup>. This string/field theory correspondence has interesting applications. When combined with Rehren's theorem, it has implications for issues concerning space-time structure and Lorentz symmetry. Our results indicate that the conjecture is correct. We are within 10-15% of the expected results, although the numerical evidence is not yet decisive.

### 1 Introduction

The so-called Maldacena conjecture<sup>2</sup>, namely the assertion that four-dimensional  $\mathcal{N}=4$  supersymmetric Yang-Mills theory ( $\text{SYM}_{3+1}$ ) can be identified in some limits with type IIB string theory on an  $AdS_5 \times S^5$  background, has caused a lot of excitement in recent years. Interesting consequences arise when this conjecture is combined with Rehren's theorem, which states that if  $\text{SYM}_{3+1}$  is an algebraic quantum field theory, it induces a dual algebraic field theory on an  $AdS_5$  space-time with fixed causal structure. As was pointed out by Arnsdorf and Smolin<sup>3</sup>, one of the following must then be true. Either  $\text{SYM}_{3+1}$  is not an algebraic quantum field theory, since it violates the causal structure of four-dimensional Minkowski space, or string theory on  $AdS_5 \times S^5$  is not a quantum theory of gravity, because it is equivalent to a quantum field theory with fixed causal structure, or there is no consistent quantization of string theory on  $AdS_5 \times S^5$  for finite string length and coupling. These findings seem to have important implications for our view on the structure of space-time and Lorentz symmetry. Since Rehren's work contains a strict proof, whereas Maldacena's conjecture is a falsifiable hypothesis, it is obvious that there is a need for a rigorous test of the latter.

In order to test the Maldacena conjecture, one would ideally have the following requirements fulfilled at the matching point of the field and string theory: small curvature to be able to work in the supergravity (SUGRA) approximation to string theory and small coupling in order to use perturbation theory on the field theory side. There is, however, no appropriate scenario known, where both requirements would be met. The way out of this dilemma is to use a non-perturbative method, namely supersymmetric discretized light-cone quantization (SDLCQ) in low dimensions, where it is known to work best. Fortunately, a scenario where a string theory corresponds to a low-dimensional field theory is available. A system of D1 branes in type IIB string theory

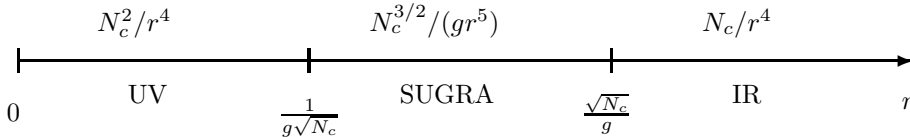
decoupling from gravity is conjectured to be dual to  $\mathcal{N} = (8, 8)$  supersymmetric Yang-Mills theory in 1+1 dimensions<sup>6</sup>. We will use the correlation function of a gauge invariant operator, namely the stress-energy tensor  $T^{\mu\nu}$ , as an observable that can be computed on both sides of the correspondence, and therefore can be used to test the Maldacena conjecture.

## 2 The Correlator from SUGRA

To determine the two-point correlation function of the stress-energy tensor from string theory, one uses the supergravity, *i.e.* small curvature approximation. We do not have room here to go into the details of the computation and refer the reader to the literature<sup>7</sup>. It suffices to state that the leading non-analytic term in the flux factor yields the correlator

$$\langle \mathcal{O}(r) \mathcal{O}(0) \rangle = \frac{N_c^{3/2}}{gr^5}. \quad (1)$$

As a consistency check we remark that in two-dimensional  $\mathcal{N} = (8, 8)$  SYM one has conformal fixed points at the ultraviolet and infrared with central charges  $N_c^2$  and  $N_c$ , respectively. One expects to deviate from the conformal  $1/r^4$  behavior of the correlator at distances  $r = 1/g\sqrt{N_c}$  and  $r = \sqrt{N_c}/g$ . This yields the phase diagram depicted below. We shall be interested in reproducing the cross-over from the small to the intermediate distance regime, where the correlator changes its behavior from  $1/r^4$  to  $1/r^5$ . The agenda is then to detect a  $1/r$  slope when evaluating the correlator at increasing distances on the field theory side.



## 3 The correlator from SDLCQ

Discrete light-cone quantization (DLCQ)<sup>8</sup> is known to preserve supersymmetry<sup>5</sup>. This makes it possible to avoid the inherent severe renormalization problems in this Hamiltonian approach to quantum field theory, given enough supersymmetry. The method goes under the name of supersymmetric DLCQ, or SDLCQ.

To reproduce SUGRA scaling relation, and to calculate the cross-over behavior of the correlator at intermediate distances using SDLCQ, we have to

compute the correlator

$$F(x^-, x^+) = \langle \mathcal{O}(x^-, x^+) \mathcal{O}(0, 0) \rangle, \quad (2)$$

where we introduced the light-cone coordinates  $x^\pm \equiv \frac{1}{\sqrt{2}}(x^0 \pm x^1)$ . As an operator we consider the gauge invariant (two-body) operator  $T^{++}(-K)$ , a component of the stress-energy tensor. In DLCQ one fixes the total longitudinal momentum,  $P^+ = K\pi/L$ , so we Fourier transform, and decompose the result into momentum modes. Finally, we continue to Euclidean space by taking distance  $r^2 = 2x^+x^-$  to be real. With the harmonic resolution  $K$  playing the role of a discretization parameter, we are supposed to send  $K \rightarrow \infty$  to recover the continuum limit. We obtain the functional form of the correlator

$$\mathcal{F}(r) = \left( \frac{x^-}{x^+} \right)^2 F(x^-, x^+) = \left| \frac{L}{\pi} \langle n | T^{++}(-K) | 0 \rangle \right|^2 \frac{M_n^4}{8\pi^2 K^3} \mathcal{K}_4(M_n r), \quad (3)$$

with the mass eigenvalues  $M_n$  and a modified Bessel function  $\mathcal{K}_4$ . Note that this result is  $K$  dependent, but involves no other unphysical quantities. In particular, the box length  $L$  will drop out. We obtain the correct small  $r$  behavior

$$\mathcal{F}(r) \longrightarrow \frac{(2n_b + n_f)}{4\pi^2} \left( 1 - \frac{1}{K} \right) \frac{N_c^2}{r^4}. \quad (4)$$

## 4 Results

The correlator, Eq. (3), is determined by numerical calculation of mass spectrum,  $M_n(K)$ , of  $\mathcal{N} = (8, 8)$  SYM. Amongst the problems we face with the numerical approach is that due to the large number of particle species in the theory, the Fock space grows very fast with the harmonic resolution:  $K = 2, 3, 4$  implies a dimension of the Hamiltonian of 256, 1632, 29056. The necessary improvements on numerical treatment include the use of a C++ code (more efficient data structure), use of the discrete flavor symmetry, and improvements on numerical efficiency (improved Lanczos algorithm). Another problem is the occurrence of massless unphysical states. The number of partons in these artifacts is even (odd) for  $K$  being even (odd). Since the correlator is only sensitive to two particle contribution, the curves  $\mathcal{F}(r)$  will show a different behavior for even and odd  $K$  in the region where the approximation breaks down. The problem is that the unphysical states yield the correct  $1/r^4$  behavior, but have a wrong  $N_c$  dependence, which prohibits the detection of the regular contribution at large  $r$ , which is down by  $1/N_c$ . We can, however, take the different behavior of the even and odd  $K$  curves to establish where the approximation breaks down. The continuum limit seems sound, because the breakdown of the approximation occurs at larger  $r$  as  $K$  grows.

Our expectations are then the following. The behavior of the correlator  $\mathcal{F}(r)$  changes like  $1/r^4 \rightarrow 1/r^5$  as  $r$  increases, so we should approach  $d\mathcal{F}/dr = -1$  in the continuum limit. Hence, we would claim success if the curve  $d\mathcal{F}/dr$  flattens at  $-1$  before the approximation breaks down. A look at Figs. 1(a) and (b), reveals that these expectations are realized. There is a clear tendency of the curves in Fig. 1(a) to develop a negative slope of order unity as  $K$  increases. To allow for a more quantitative statement, we plotted the derivative of the curves in Fig. 1(b). Keeping in mind that our approximation breaks down when the odd and even  $K$  curves cross, we see that the values of the correlator at this point seem to converge towards unity, as  $K$  grows.

## 5 Conclusions and Outlook

In order to test the Maldacena conjecture, we calculated the correlator of the stress-energy tensor on the field theory side of this string/field theory correspondence with the non-perturbative SDLCQ approach. Our results are within 10-15% of the results expected from Maldacena conjecture. The present study includes a factor 100-1000 more states than previously considered<sup>4</sup>. Improvements of code and the numerical method are possible and are either on the way, or have been implemented already. Empirically, we found that the contributions to the matrix elements come from small number of terms. An analytic understanding of this fact would greatly accelerate calculations and help improving the test of the conjecture.

As an outlook, we state that for the final goal, namely to test the Maldacena conjecture proper ( $\mathcal{N} = 4$  SYM<sub>3+1</sub> vs. type IIB string theory on  $\text{AdS}_5 \times S_5$ ), we have to apply the SDLCQ approach in larger dimensions. This has been partly achieved already in a series of papers of Pinsky, Hiller, and the author<sup>10,9</sup> with novel numerical improvements by orders of magnitude.

1. S. Pinsky, O. Lunin, J. Hiller, U. Trittman, *Phys. Lett.* **B482** 409.
2. J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231.
3. M. Arnsdorf and L. Smolin, hep-th/0106073.
4. F. Antonuccio, O. Lunin, S. Pinsky, A. Hashimoto, JHEP **07** (1999) 029.
5. Y. Matsumura, N. Sakai, and T. Sakai, *Phys. Rev.* **D52** (1995) 2446.
6. N. Itzhaki, J. Maldacena, J. Sonnenschein, and S. Yankielowicz, *Phys. Rev.* **D58** (1998) 046004.
7. A. Hashimoto and N. Itzhaki, *Phys. Lett.* **B465** (1999) 142–147.
8. S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky, *Phys. Rept.* **301** (1998) 299.
9. J.R. Hiller, S. Pinsky, U. Trittman, *Phys. Rev.* **D63** (2001) 105017.
10. J.R. Hiller, S. Pinsky, U. Trittman, hep-th/0106193, to appear in *Phys. Rev. D*.

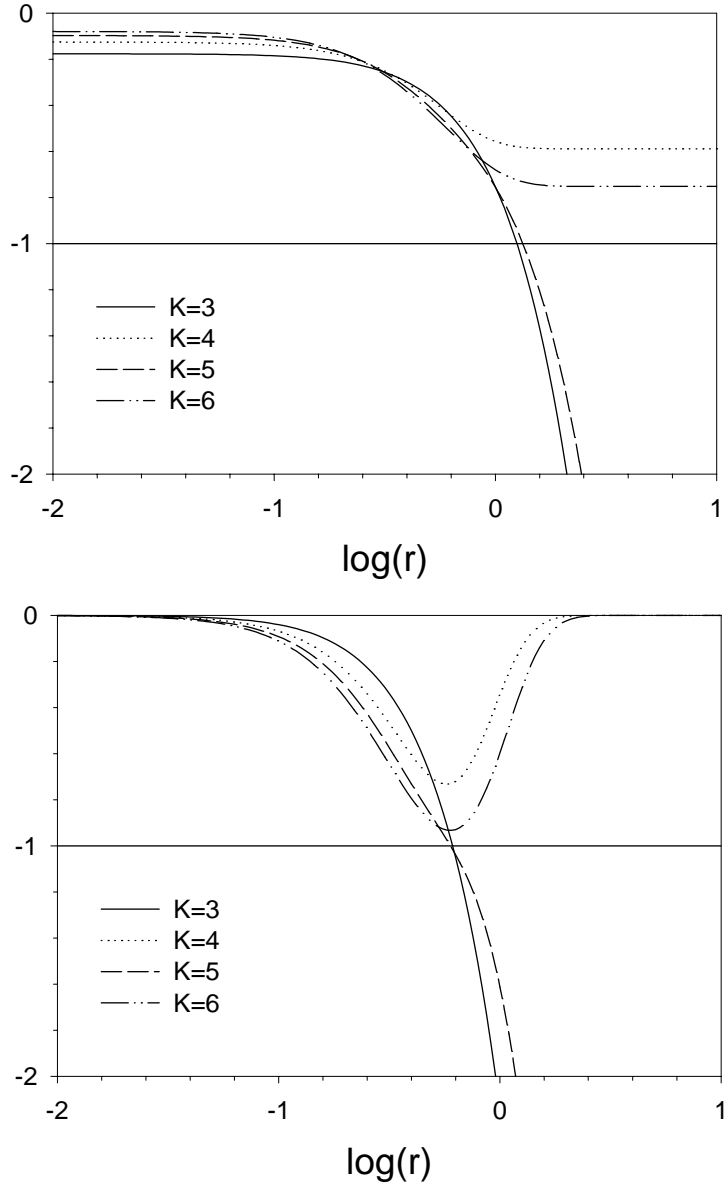


Figure 1: (a) Top: log-log plot of correlator  $\langle T^{++}(x)T^{++}(0) \rangle \left( \frac{x^-}{x^+} \right)^2 \frac{4\pi^2 r^4}{N_c^2(2n_b+n_f)}$  v.s.  $r$  in units  $g^2 N_c / \pi$  for  $K = 3, 4, 5$  and  $6$ . (b) Bottom: the log-log derivative with respect to  $r$  of the correlation function in (a).